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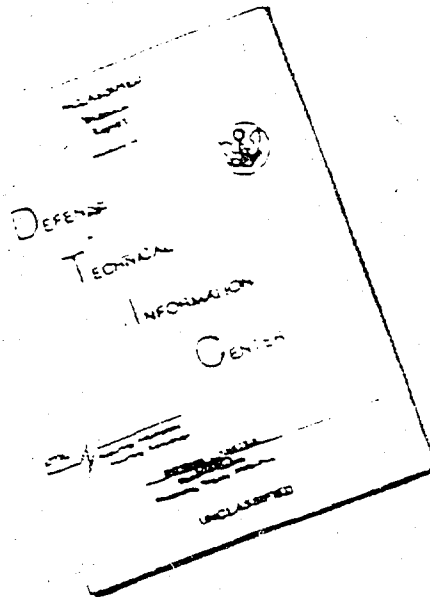
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A BIBLIOGRAPHY OF SEARCH THEORY AND RECONNAISSANCE THEORY LITERATURE

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ABSTRACT

This bibliography consists of 75 items on search theory and reconnaissance theory found in periodicals and technical reports. Each entry includes a short summary or a quotation from the pertinent abstract. The items are classified into one or more of the following categories: (1) General Discussion of Problems; (2) Allocation of Effort; (3) Game Theory Formulations; (4) Geometric Search Patterns; (5) Measures of Performance and (6) Miscellaneous.

A. INTRODUCTION

Search theory was one of the earliest problems studied by the operations research groups in the United States. During World War II a large amount of work was done in this area by the organization now known as the Operations Evaluation Group of the Office of the Chief of Naval Operations, Department of the Navy. A major motivating factor in their work was the requirement for better tactics for antisubmarine warfare operations. The results of this early work are summarized in the publications of Bernard O. Koopman.

Even with this early start, the field still appears to be relatively unstructured. Specific problems have been examined and some solutions obtained, but very little general theory has evolved. The categories used to classify the entries in this bibliography are based on what appear to be the natural divisions of the publications, not necessarily natural divisions of the subject matter.

Search theory and reconnaissance theory represent studies of certain classes of information-gathering problems. Originally, search theory related to searching physical space for tangible objects, such as enemy submarines, aircraft, etc.; reconnaissance theory was concerned with pure information-gathering problems. When search problems are examined from the point of view of statistical decision theory, however, there is no basic difference between the two classes of problems. The fundamental purpose of all such operations is to obtain information about the location of the target.[†]

The aspect of this problem that has received the most attention is the allocation of the available search effort over the space to be searched. Most of the studies in this area have examined the effect of different payoff functions on the allocation (e.g., how does the

*Major, U.S. Army.

†"Target" refers to the object of the search whether it be a military target, a mineral deposit, or any other object about which information is desired.

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probability of success increase with increases in the effort allocated to a subspace?). In some studies the search effort is divisible only into discrete elements, while in others it is continuously divisible.

The optimum allocation of effort is dependent on what criterion is used to measure the performance of the search operations. Most authors studying allocation of effort have ignored the existence of different measures, although a few papers have been written in this area.

An aspect of the problem closely related to the allocation of effort is geometrical search patterns. The two main subdivisions of this area are:

1. Patterns used to conduct general search operations over the entire space (e.g., a plane making parallel passes over the area to be searched).
2. Patterns used in special geometrical situations (e.g., a man in the ocean knows that he is one mile from the shore, but he does not know its direction. What path should he swim to be sure of reaching the shore in the minimum time?).

There has been some work on the use of game theory techniques to examine search problems, particularly search operations against noncooperating targets that may make efforts not to be found. As with most uses of game theory, these studies have primarily resulted in a framework that aids in the analysis of the results of possible combinations of action by the searcher and the evader.

B. COVERAGE AND AVAILABILITY OF REFERENCES

The search for items to include in this bibliography has covered many sources. Primary among these were personal correspondence with known workers in the field and references cited in other works. The source of each publication is cited, and if it is known to be available from the Defense Documentation Center, its "AD" number is also given. The largest single source is the Operations Evaluation Group either directly or through contract work.*

Readers having knowledge of other sources that should be included in this bibliography are invited to send this information to the author in care of Stanford Electronics Laboratories.

C. CROSS-REFERENCE INDEX

The items in this bibliography have been divided into six categories which are based primarily on the material covered. Several items are cited under more than one category in the cross-reference index given below.

1. General Discussion of Problems

Ackoff, 1956

Ackoff, 1961

Ackoff and Rivett, 1963

Langendorf, 1959

McDonald, Fergusson, and Elliott, 1962

Morse, 1948

Morse and Kimball, 1951

*The one area of possible sources that has not been searched carefully is the literature on mineralogical surveys.

2. Allocation of Effort

- | | |
|-----------------------------|--|
| Arnold, 1962 | Gluss, 1959 |
| Blachman, 1959 | Gluss, 1961 |
| Blachman and Proschan, 1959 | Gumacos, 1963 |
| Black, 1962 | Karchere and Hoeber, 1953 |
| Bram, 1963 | Koopman, 1946 |
| Charnes and Cooper, 1958 | Koopman, 1953 |
| Danskin, 1960 | Koopman, 1954 |
| Danskin, 1962b | Koopman, 1957 |
| Danskin, 1964 | Koopman, 1959 |
| Danskin, 1965a | McDonald, Fergusson, and Elliott, 1962 |
| Danskin, 1965b | MacQueen and Miller, 1960 |
| de Guenin, 1959 | Matula, 1964 |
| de Guenin, 1961 | Neuts, 1963 |
| Dobbie, 1963 | Norris, 1962 |
| Dobbie, 1964 | Pollock, 1960 |
| Engel, 1957 | Pollock, 1964 |
| Enslow, 1965 | Posner, 1963 |
| Fine, 1946 | Potter, 1961 |
| Firstman and Gloss, 1960 | Scott, 1963 |
| Giammo, 1963 | Zahl, 1963 |
| Gilbert, 1959 | |

3. Game Theory Formulations

- | | |
|---------------------------|---------------|
| Arnold, 1962 | Dubins, 1957 |
| Belzer, 1949 | Isaacs, 1965 |
| Bram, 1963 | Johnson, 1934 |
| Danskin, 1960 | Neuts, 1963 |
| Danskin, 1962b | Norris, 1962 |
| Danskin and Gillman, 1953 | Sherman, 1949 |
| Dresher, 1961 | |

4. Geometric Search Patterns

- | | |
|----------------|---------------------------|
| Alderson, 1954 | Kimball, 1946a |
| Fine, 1946 | Kimball, 1946b |
| Gluss, 1961a | Koopman, 1946 |
| Gluss, 1961b | Koopman, 1956a |
| Gluss, 1961c | Koopman, 1957 |
| Gross, 1955 | Koopman, 1959 |
| Isaacs, 1965 | Nichols and Whisler, 1961 |
| Isbell, 1957 | Posner, 1963 |

5. Measures of Performance

Barlow, 1960	Kimball, 1946b
Danskin, 1959	Kobzarev and Basharinov, 1961
Danskin, 1962a	Koopman, 1946
Enslow, 1965	Mela, 1961
Fine and Lamb, 1947	Novosad, 1961
Giammo, 1963	

6. Miscellaneous

Bram and Weingarten, 1964	Koopman, 1956a
Hunt, 1963	Koopman, 1956b
Kimball, 1963	Potter, 1960

D. ALPHABETICAL LISTING

This section lists the 75 items comprising the bibliography proper. In addition to being listed alphabetically by author, they are numbered sequentially for convenience in cross-referencing and ready identification. The pertinent categories into which each item has been classified are shown immediately below the source citation.

1. Ackoff, Russell L., **THE DEVELOPMENT OF OPERATIONS RESEARCH AS A SCIENCE**, Operations Research, 4:3, pp. 265-295, June 1956.

Category: General Discussion

Mentions search theory as developed during World War II. Search problems form a subclass of information-collection processes.

2. Ackoff, Russell L. (ed.), **PROGRESS IN OPERATIONS RESEARCH**, Vol. I, John Wiley & Sons, New York, 1961.

Category: General Discussion

Merely cites Ackoff's 1956 paper (item 1), but does indicate that search problems have now risen in stature to a class by themselves.

3. Ackoff, Russell L., and Patrick Rivett, **A MANAGER'S GUIDE TO OPERATIONS RESEARCH**, John Wiley & Sons, New York, 1963.

Category: General Discussion

Classifies search problems as one of eight forms of operations research problems. Discusses kinds of errors which can be made: (1) "sampling error" — failing to look in the proper location at the proper time, and (2) "observational error" — failing to detect the target object even though it is being looked at, or giving a false alarm when it is not there. Discusses the nature of the constraints on the problem and some application made of the results — auditing, quality control, etc. Mentions inverse problem of retailer displaying goods, counters, etc., to match search patterns of customers.

4. Alderson, Wroe, **SEARCH THEORY IN THE ANALYSIS OF CONSUMER SHOPPING**, Third National Meet of the ORSA, Boston, Mass., 23-24 Nov 1953, Invited paper, Abstract in Operations Research, 2:1, p. 78, Feb 1954.

Category: Geometric Search Patterns

"Marketing may be regarded in one of its aspects as a double search in which producers are looking for consumers who can use their products, and consumers are looking for goods which will satisfy their needs." (Actual paper not reviewed.)

5. Arnold, Robert D., AVOIDANCE IN ONE DIMENSION: A CONTINUOUS-MATRIX GAME, Operations Evaluation Group, Office of Chief of Naval Operations, Washington, D.C., OEG IRM-10 (AD 277 843), 14 pp., 11 Jan 1962.

Category: Allocation of Effort
Game Theory

"The game is a two-person zero-sum game. On each play, each player selects any point on a line of finite length. The payoff is a trapezoidal function of the separation between the two selected points; it is constant for separations from zero to R_1 and R_2 , changes linearly between R_1 and R_2 , and is zero for separations greater than R_2 . The derivation and proof of the solution are interesting due to the discontinuities in the slope of the payoff function. The solution includes the special cases of triangular ($R_1 = 0$) and rectangular ($R_1 = R_2$) payoff functions. The game is related to search theory in its applicability to the barrier problem. Uniform distribution along the barrier is not, in general, an optimal strategy for either the maximizer (detector) or the avoider (transitor). In selecting optimal strategies the detector must have more information on the payoff function (lateral range curve) than is required by the transitor."

6. Barlow, Richard, PROBABILITY PROBLEMS IN DETECTING MISSILE SALVOS, Electronic Defense Laboratories, Sylvania Electric Products, Inc., Mountain View, Calif., EDL-M257 (AD 255 667), 31 pp., 12 Feb 1960.

Category: Measures of Performance

"Various models describing missile arrivals resulting from a salvo attack are discussed. The effectiveness of detecting equipment which has an operative delay after each missile detection is considered. The probability of 'seeing' a given percentage of the missile salvo is obtained for random missile arrivals. The distribution of the time lag in generating a 'warning' is obtained for detection equipment with a given detection reliability."

7. Belzer, R. L., SOLUTIONS OF A SPECIAL RECONNAISSANCE GAME, RAND Corporation, 1700 Main St., Santa Monica, Calif., RM-203, 23 pp., 10 Aug 1949.

Category: Game Theory

"In this paper, solutions are given for a special case of a general reconnaissance model previously investigated by S. Sherman. . . .

"The game investigated here permits player I to reconnoiter at a cost and player II to take certain countermeasures for which he also pays a price. It is interesting to notice that the strategies of player I do not depend on the price of reconnaissance and the strategies of player II do not depend on the cost of countermeasures. We conjecture that this is true for the $2m \times 2n$ matrix, R . We also suspect that if a chance mechanism were used to determine if, and how much, information should be given to player I about player II's move, in the event both reconnaissance and countermeasures were used, the disparity between the numbers of optimal strategies available to each player, which is so noticeable here, might be removed. Except for a small range of values of the costs of employment of reconnaissance and countermeasures, the value of the game with reconnaissance does not

depend on the cost of countermeasures. This perhaps indicates that the solutions of the game are not significant outside of these ranges from the point of view of interest in the model. In these ranges, both players have a unique optimal strategy and the value of the game depends both upon the cost of reconnaissance and the cost of countermeasures. The optimal strategies and values of the game for the various cases are summarized in tabular form at the end of this paper."

8. Blachman, Nelson, PROLEGOMENA TO OPTIMUM DISCRETE SEARCH PROCEDURES, Naval Research Logistics Quarterly, 6:4, pp. 273-281, 1959.

Category: Allocation of Effort

"An object may appear in any one of n locations and, having appeared, will remain there. The probability of appearance in the i th location is p_i , and the time of appearance is distributed uniformly over a long interval. A look in the i th location takes a time t_i and it detects the object with probability P_i if it is there. A search procedure, i.e., a sequence of locations in which to look, is desired which minimizes the expected delay between the appearance and the detection of the object."

9. Blachman, Nelson, and Frank Proschan, OPTIMUM SEARCH FOR OBJECTS HAVING UNKNOWN ARRIVAL TIMES, Operations Research, 7:5, pp. 625-638, Sep-Oct 1959.

Category: Allocation of Effort

"Objects arrive in accordance with a Poisson process. Having arrived, the object appears (and remains until detected) in box i with probability p_i . A single scan of box i costs C_i (possibly including the cost of false alarms), takes time t_i , and if the object is present in box i at the beginning of the scan, will detect with probability P_i . The resultant gain $g_i(t)$ is a nonincreasing function of t , the delay between arrival and the beginning of the detecting look; $i = 1, 2, \dots, n$. An asymptotically optimum search procedure is obtained. A number of particular cases of interest are solved."

10. Black, William Lawrence, SEQUENTIAL SEARCH, Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass., Master of Science Thesis, June 1962.

Category: Allocation of Effort

"This paper discusses searches for a single stationary object, hidden in one of several regions. The single searcher cannot examine more than one region at a time, hence is forced to examine them sequentially. Even if the searcher looks in the correct region, it is possible that he overlooks the object. The problem is to determine the search policy that 'minimizes' in some sense the cost of the search.

"In discrete, or quantized effort searches, we show that often several criteria of optimality lead to the same search policy. Even when this circumstance does not prevail, we show how to find the policy minimizing the expected cost of the search.

"We also show how to minimize expected cost in a continuous two region search where there is a fee for switching effort from one region to the other. This problem requires more sophisticated techniques than the discrete searches treated."

11. Bram, Joseph, A 2-PLAYER N-REGION SEARCH GAME, Operations Evaluation Group, Office of Chief of Naval Operations, Washington, D.C., OEG IRM - 31 (AD 402 914), 21 pp., 17 Jan 1963.

Category: Allocation of Effort
Game Theory

"Given N regions with their associated conditional detection probabilities $\alpha_1, \dots, \alpha_N$, let player A choose one region to hide in, and let player B look in one region at a time until he finds A. The payoff, to player A, is the expected number of looks required of B to find A. The form of the optimal pure strategies for B is described, and the mixed extension of this game is shown to have a solution. Player B has a good strategy that is a mixture of at most N pure strategies. A numerical procedure for calculating the solution is given."

12. Bram, J. and H. Wellington, ESTIMATION OF BINOMIAL PARAMETERS FROM SEARCH DATA, Center for Naval Analysis, Washington, D.C., Res Contribution No. 3, 7 pp., 5 May 1964.

Category: Miscellaneous

The parameters of the binomial distribution are the probability of success, p , and the number of trials.

"During a search we assume that there are a fixed but unknown number of objects, m , available to be detected. We assume that the probability of detecting an object is an unknown constant p . We repeat the search k times. . . . We record after each search only the new detections made during that search. We then have k random variables,

$$d_j, j = 1, 2, \dots, k$$

where d_j is the number of new detections made during the j -th search. We want to estimate p and m from the observed values of d_j ."

The answers are obtained by the solution of the simultaneous equations:

$$\hat{m} = \frac{S}{1 - q^k}$$

$$\hat{p} = \frac{S}{k(m - S) + T}$$

where

$$S = \sum_{j=1}^k d_j$$

$$T = \sum_{j=1}^k j d_j$$

13. Charnes, A. and W. W. Cooper, **THE THEORY OF SEARCH: OPTIMUM DISTRIBUTION OF SEARCH EFFORT**, Management Science, 5:1, pp. 44-50, Oct 1958.

Category: Allocation of Effort

"This paper will be concerned with formulating the optimum allocation of search effort as a problem in convex programming so that their solutions may be made amenable to treatment by the adjacent extreme point methods of linear programming. Attention will be concentrated on discrete (statistical) distributions because this class of cases admits of the easiest and most straight-forward treatment. A sketch will then be given of how extensions may also be effected to continuous distributions."

14. Danskin, John M., **THE THEORY OF RECONNAISSANCE**, Wright Air Development Center. Wright-Patterson AFB, Ohio, WADC Tech Note 59-409 (AD 233 998), 31 pp., Nov 1959.

Category: Measures of Performance

Almost identical to the paper "A THEORY OF RECONNAISSANCE: I" by the same author [item 16].

15. Danskin, John M., **THE THEORY OF RECONNAISSANCE**, Aero Research Lab., AF Research Division, ARDC, Wright-Patterson AFB, Ohio, ARL Tech Report 60-337 (AD 254 108), 20 pp., Dec 1960.

Category: Allocation of Effort

Game Theory

Almost identical to the paper "A THEORY OF RECONNAISSANCE: II" by the same author [item 17].

16. Danskin, John M., **A THEORY OF RECONNAISSANCE: I**, Operations Research, 10:3, pp. 285-299, May-June 1962a.

Category: Measures of Performance

"A study is made of the optimum distribution of serial reconnaissance effort against land targets in the presence of decoys. The model considered is one in which the reconnoitering forces allocate effort among various regions, their objective being the location of the targets, assuming that the side being reconnoitered is passive. The information function of the theory of communication is chosen as the measure of effectiveness. It is proved that under certain reasonable assumptions the information function is increasing and convex-concave. Thus the problem is reduced to a familiar one in operations research, the maximization of the sum of several convex-concave functions subject to linear side conditions."

17. Danskin, John M., **A THEORY OF RECONNAISSANCE: II**, Operations Research, 10:3, pp. 300-309, May-June 1962b.

Category: Allocation of Effort

Game Theory

"This part concerns the two-sided reconnaissance problem, in which the side being reconnoitered seeks to minimize the information (maximize the confusion) obtained by the reconnoiterer, while maintaining at least a certain minimum acceptable threat with a fixed budget. This problem, formulated as a zero-sum, two-person game, is solved for

one special case (fixed equipment) and it is proved that there exists a solution in mixed strategies for the general case."

18. Danskin, John M., ON KOOPMAN'S ADDITION THEOREM IN SEARCH THEORY, Institute of Naval Studies, Cambridge, Mass., 1964a (to be published in Operations Research).

Category: Allocation of Effort

"Koopman conjectured some years ago that his addition theorem, which states that the sum of a maximizing strategy using effort A and a maximizing strategy using effort B on the a posteriori distribution left by A is maximizing for the effort A + B, was a consequence of the exponential character of his effectiveness function. This paper shows that this conjecture is exactly correct, that any effectiveness function having this property is for exponential type. It also gives conditions for two looks to be better, the same, or worse than one. As a consequence one finds that the expanding spiral is not the best two-dimensional search pattern for a fixed circularly distributed object except in the special cases."

19. Danskin, John M., THE THEORY OF MAX-MIN, WITH APPLICATIONS, Institute of Naval Studies, Franklin Institute, Cambridge, Mass., 1965a (to be published in the Journal of the Society of Industrial and Applied Mathematics).

Category: Allocation of Effort

"The basic tool, studied in Part I, is a new kind of directional derivative. Let $F(x, y)$ be a function with $x = (x_1, \dots, x_n)$ in Euclidean space and y in any compact topological space. Suppose $F(x, y)$ and its partial derivatives $F_{x_i}(x, y)$ are continuous. Put

$$(*) \quad \varphi(x) = \min_y F(x, y).$$

In operations research the problem of interest is to maximize $\varphi(x)$. Unfortunately, in the simplest problems, with smooth $F(x, y)$, $\varphi(x)$ is not differentiable in the usual sense. Characteristically it has sharp ridges. It turns out that nevertheless $\varphi(x)$ has a directional derivative in each direction, given by an explicit formula: if $\gamma_1, \dots, \gamma_n$, with $\gamma_1^2 + \dots + \gamma_n^2 = 1$, represents a direction in the n -dimensional space of x , then

$$(**) \quad D_\gamma \varphi(x) = \min_{y \in Y_x} \sum \gamma_i F_{x_i}(x, y),$$

the minimum being taken over the set Y_x of all those y 's which yield the minimum against x in (*). Corners and sharp ridges in $\varphi(x)$ occur only when Y_x consists of more than a single point. The derivative $D_\gamma \varphi(x)$ does not satisfy the usual addition formulas for derivatives: if $\gamma = a\alpha + b\beta$ for two directions α and β , and a and b are positive, we have

$$(***) \quad D_\gamma \varphi(x) \geq a D_\alpha \varphi(x) + b D_\beta \varphi(x).$$

With equality, this would be the usual addition formula which allows one to deal with the ordinary directional derivative by taking linear combinations of the derivatives along the axes. However, we present a simple example in which $D_\alpha \phi(x) = 0$, $D_\beta \phi(x) = 0$, and $D_\gamma \phi(x) > 0$. Thus the derivative $D_\gamma \phi(x)$ is a new object; it cannot be so reduced; the derivatives along the axis are not enough.

"This derivative is applied in Part II to study the problem (not a game!)"

$$\text{Max}_x \text{ Min}_y \sum v_i \left(1 - \alpha_i e^{-k_i x_i / Y_i} \right)^{Y_i},$$

subject to $\sum x_i = X$, $\sum y_i = Y$, $x_i \geq 0$, $y_i \geq 0$. The criteria as to whether the x_i should be positive in the solution are found. The applications to economics are discussed in Part III."

20. Danskin, John M., A MAX-MIN PROBLEM ASSOCIATED WITH STRATEGIC DETERRENCE THEORY, Institute of Naval Studies, Franklin Institute, Cambridge, Mass., 1965b (to be published in the Naval Research Logistics Quarterly).

Category: Allocation of Effort

"There have appeared in recent years a number of problems in allocation theory which can be treated neither by traditional economic methods nor by standard game theory.

"These are problems with the superficial features of a game, in which there is a 'payoff' function $F(x, y)$, with the x controlled by one 'player' and the y by another. The x may be an allocation (x_1, \dots, x_n) subject to side conditions, such as $\sum x_i = 1$, $x_i \geq 0$, and similarly for the y . The x -player wishes to maximize and the y -player to minimize.

The problem

We are concerned with the problem

$$\text{Max}_x \text{ Min}_y \left[\sum_{i=1}^n v_i x_i e^{-\alpha_i y_i} + \sum_{j=n+1}^m v_j x_j e^{-\alpha_j y_j / x_j} \right]$$

subject to

$$\sum v_i + \sum x_i = 1, x_i, x_j \geq 0$$

and

$$\sum y_i + \sum y_j = 1, y_i, y_j \geq 0$$

The $v_i, v_j, \alpha_i, \alpha_j$ are supposed positive.

"This problem is not a game. The Min Max exceeds the Max Min in many cases.

"We obtain a complete description of the solution of this problem in a form which has successfully been applied to both computer and hand calculations in specific cases."

21. Danskin, J. M. and L. Gillman, A GAME OVER FUNCTION SPACE, Revista di Matematica della Universita di Parma, Vol. 4, 1953.

Category: Game Theory

(A copy of this paper could not be located.)

22. de Guenin, J., LES FONDEMENTS D'UNE THEORIE DE LA RECHERCHE, Revue de Statistique Appliquee, 7:4, 1959.

Category: Allocation of Effort

This same article appeared in the Journal of Operations Research, 9:1, pp. 1-7, Jan-Feb 1961; [item 23].

23. de Guenin, Jacques, OPTIMUM DISTRIBUTION OF EFFORT: AN EXTENSION OF THE KOOPMAN BASIC THEORY, Operations Research, 9:1, pp. 1-7, Jan-Feb 1961.

Category: Allocation of Effort

"The fundamental problem of search theory is to allocate a given amount of search effort in such a way as to maximize the overall probability of discovering an object located in a given space. The problem has already been solved under the assumption that the probability of detecting the object is a negative exponential function of the search effort density; this assumption seems to be fairly realistic in many military applications, but has definite drawbacks in others. In this paper a method is provided for solving the problem in the general case, where no assumption is made concerning the form of the detection probability function. A theorem is derived that gives a general relation governing the optimal solution."

24. Dobbie, James M., SEARCH THEORY: A SEQUENTIAL APPROACH, Naval Research Logistics Quarterly, 10:4, pp. 323-334, Dec 1963.

Category: Allocation of Effort

"Koopman and others have noted that the optimal distribution of effort, to maximize the detection probability with a given effort, has the property that it is the sum of the distributions obtained by optimizing conditionally when the effort is applied sequentially. The approach taken in this present investigation is to reverse the procedure by determining conditions under which the optimal distribution has this property, and then to use this property to find the optimal distribution. This method easily yields essentially all of the known results, both for the discrete and continuous cases, and for the two criteria of maximizing the detection probability with a given effort and minimizing the expected effort to attain a given attainable probability of detection. The method also yields some modest extensions. In the discrete case the method is equivalent to the simplest form of dynamic programming."

25. Dobbie, James M., SURVEILLANCE OF A REGION BY DETECTION AND TRACKING OPERATIONS, Operations Research, 12:3, pp. 379-394, May-June 1964.

Category: Allocation of Effort

"A study is made of the capabilities of a surveillance system to detect and track submarines that are in a region that is under surveillance. The operation consists of barrier searches for submarines entering the region, area searches for submarines that have

entered the region undetected, tracking procedures to hold contact on detected submarines, and special searches to regain contact when contact has been lost. The capabilities of the surveillance system are found for a general distribution of submarine on-station times, under the assumption that the recontact rate decreases with increasing time after loss of contact."

26. Dresher, Melvin, **GAMES OF STRATEGY - THEORY AND APPLICATION**, Prentice-Hall, Englewood Cliffs, N. J., pp. 61-68, 175-178, 1961.

Category: Game Theory

Treats the investigation of the advisability of reconnaissance before an attack as a game of strategy. Analysis assumes costs for obtaining information and allows enemy to use countermeasures. This type of analysis has obvious application to the design of the employment plan for an observation system. Also considered is the infinite game formulation of a tactical reconnaissance problem. The value of the target is unknown; losses are possible for both reconnaissance and bomber aircraft; and knowledge of the target value will permit dispatch of most efficient size task force. How many aircraft should be sent on reconnaissance, and, based on their report (or failure to return), what size attack force should be dispatched? This latter discussion is an excellent example of the total systems concept of the role played by an observation system.

27. Dubins, L. E., **A DISCRETE EVASION GAME**. Annals of Mathematics Studies, No. 39, "Contributions to the Theory of Games, Vol. III," (eds.) M. Dresher, A. W. Tucker, and P. Wolfe, Princeton University Press, Princeton, N. J., pp. 231-255, 1957.

Category: Game Theory

The searcher takes action (i.e., drops a bomb) at time 0, but it requires k units of time before this action can affect the target. During the k units of time, the target can take evasive actions.

28. Engel, J. H., **USE OF CLUSTERING IN MINERALOGICAL AND OTHER SURVEYS**, Proc. of the First International Conference on Operations Research, Oxford, 1957, The English Universities Press, pp. 176-192.

Category: Allocation of Effort

"This paper considers the solution of a two-stage search problem. The first stage or preliminary search is imperfect; it misses some of the prizes and occasionally produces spurious signals where no prize is present. The second stage or detailed search finds the prizes if they are there, but is expensive, difficult or time consuming, so that it is concentrated only in areas likely to be profitable. The problem is solved by employing a 'clustering technique' to interpret results obtained during several coverings of the area during the first-stage search. Effectiveness measures for choosing an optimal balance of effort between first and second-stage search are provided. Input parameters for such problems are defined, and illustrative examples provided."

29. Enslow, P. H., Jr., **OBSERVATION SYSTEMS EMPLOYING PERIODIC SAMPLING**, Rept. SEL-65-038 (TR No. 1906-1), Stanford Electronics Laboratories, Stanford, Calif., June 1965.

Category: Allocation of Effort

Measures of Performance

Examines measures of performance used to evaluate discrete search operations. The three measures studied are the probability of unequivocally locating the target, the probability of detecting the target ("guessing" its correct location), and the change in entropy of target location probabilities (decrease in uncertainty as to location). The distributions of search effort to maximize each of these three measures are derived, and it is shown that the three optimum distributions may all be different.

30. Fine, Nathan J., PRELIMINARY STUDY OF THE INTERCEPTION PROBLEM, Operations Evaluation Group, Office of the Chief of Naval Operations, OEG Study No. 301 (ATI 28 837), 9 pp., 27 Dec 1946.

Category: Allocation of Effort

Geometric Search Patterns

"A study was made to evaluate standard search and interception procedures suitable for scouting, and to derive methods for choosing the optimum search plan for any given situation. A derivation of a simple formula for evaluating a search from ahead is given. Plans are devised for use in searching for a target passing through a narrow channel, for a target which is known to be located somewhere within a particular area, and for other specialized search problems. The probability of sighting the target is discussed from a knowledge of sweep-width, ship-spacing, and other similar factors which are independent of the target's probability distribution."

31. Fine, Nathan J. and Frank W. Lamb, A RELATION BETWEEN THE AVERAGE RANGE OF DIRECT APPROACH AND SWEEP WIDTH, Operations Evaluation Group, Office of the Chief of Naval Operations, Washington, D.C., OEG Study No. 325 (ATI 28 829), 14 pp., 24 June 1947.

Category: Measures of Performance

"Using a simplified treatment of the probability of target detection in aerial radar sweeps, a relation is developed between \bar{F} , the average range of first detection on direct approach, and W , the sweep width, in terms of a coefficient of detection k . An expression relating these parameters simplifies sweep width determinations by requiring only operational data on average range. Since the average range on direct approach does not differ greatly from the average range of first contact on any bearing, the relationship permits the estimation of sweep widths by results obtained in any operational flying. Against large targets where k is large, it is shown that $W = (2.3) (\bar{F})$ within about 10 percent."

32. Firstman, Sidney I., and Brian Gluss, SEARCH RULES FOR AUTOMATIC FAULT DETECTION, RAND Corporation, Santa Monica, Calif., RM-2514, 31 pp., 15 Jan 1960.

Category: Allocation of Effort

"In this memorandum the development of search rules will be directed specifically toward an automatically sequenced testing machine that uses a predetermined program (i.e., a program that, once begun, will not change)... This model is specialized for (a) programs that once designed are fixed, and for (b) control equipment that has a fixed mode of operation. It is assumed that the testing will proceed, in a sequential manner based on go-no go (good or no good) results, until either the fault is found, or the machine

reaches an arbitrary intermediate stopping point with no fault located. The effects of these and other constraints or specializations upon the mathematical exposition and results will be discussed. The consideration of other types of equipment or use could lead to different results."

33. Giammo, T. P., ON THE PROBABILITY OF SUCCESS IN A SUDDEN DEATH SEARCH WITH INTERMITTENT MOVES CONFINED TO A FINITE AREA, SIAM Rev., 5:1, pp. 41-51, Jan 1963.

Category: Allocation of Effort
Measures of Performance

Analyzes the situation where two opposing mobile battle forces that are able to be moved at fixed time intervals are searching for the enemy's position, and finds the probability of detecting the enemy before being detected by him. An exact solution for the problem of detecting the enemy before being detected by him is found. It is shown that this solution is not practical analytically and a suitable approximation is therefore developed.

34. Gilbert, E. N., OPTIMAL SEARCH STRATEGIES, J. Soc. Indust. Appl. Math., 7:4, pp. 413-424, Dec 1959.

Category: Allocation of Effort

"The searcher will face one of the following situations:

- (1) Only place 1 contains the items sought
- (2) Only place 2 contains the items sought
- (3) Both places contain the items sought

The searcher will not know which one of (1), (2), or (3) he has, but he knows their a priori probabilities to be a_1 , a_2 , and a_3 with $a_1 + a_2 + a_3 = 1$."

The model considers nonzero time required to switch from one place to another.

35. Gluss, Brian, AN OPTIMUM POLICY FOR DETECTING A FAULT IN A COMPLEX SYSTEM, Operations Research, 7:4, pp. 468-477, July-Aug 1959

Category: Allocation of Effort

"Consider a complex system consisting of N modules containing $n(1)$, ..., $n(N)$ items or sub-circuits. . . . Assuming two different possible models, equations are developed for the optimum policy, i.e., the policy which minimizes the expected amount of time consumed or penalties paid, and these equations are solved for the first model. Model I assumes that overall tests of each module may be performed, and individual item tests within modules; model II assumes that overall module tests are not possible, and that penalty costs must be paid whenever the search moves from one module to another. In both models, penalty costs — in time, for example — are associated with the testing of each item and module."

36. Gluss, Brian, APPROXIMATELY OPTIMAL ONE-DIMENSIONAL SEARCH POLICIES IN WHICH SEARCH COSTS VARY THROUGH TIME, 19th National Meeting of the ORSA (Abstract in Bull. of the ORSA, Suppl. 1, Operations Research Vol. 9, 1961) (Paper revised Jan 1961a; 11 pp.)

Category: Allocation of Effort
Geometric Search Patterns

"The a priori probabilities of the object being in cells $1, \dots, N$ are p_1, \dots, p_N , respectively, and the costs of examination of these cells are t_1, \dots, t_N , respectively; the search policy is considered to be optimal when the statistical expectation of the total cost of the search is minimized. . . . It is assumed that the costs comprise a travel cost dependent upon the distance from the last cell examined, in addition to a fixed examination cost: initially, assuming that the searcher is next to cell 1, $t_1 = i + t$, where t is constant; and from then onwards, assuming that the j^{th} cell has just been examined, $t_j = |i - j| + t$. An optimal search strategy is found in the case where the p_i 's are all equal, and approximately optimal strategies in the case where p_i is proportional to i ."

37. Gluss, Brian, AN ALTERNATIVE SOLUTION TO THE "LOST AT SEA" PROBLEM, Naval Research Logistics Quarterly, 8, pp. 117-121, 1961b.

Category: Geometric Search Patterns

"A problem posed by Bellman and considered by Isbell is as follows: Suppose one is a mile from a straight shore with no means whatsoever of ascertaining its direction. What is the optimum path to follow so as to (a) minimize the maximum distance travelled in reaching shore, (b) minimize the statistical expectation of the distance travelled, or (c) maximize the probability of reaching shore within a given distance travelled? Isbell found the solution to (a); in the present paper a sequence of approximations to the optimal policy for (b) is considered."

38. Gluss, Brian, THE MINIMAX PATH IN A SEARCH FOR A CIRCLE IN A PLANE, Naval Research Logistics Quarterly, 8, pp. 357-360, 1961c.

Category: Geometric Search Patterns

"The problem has been considered by Isbell of determining the path that minimizes the maximum distance along the path to a line in the same plane whose distance from the starting point of the search is known, while its direction is unknown. We consider in this article the analogous problem for the search for a circle of known radius, of known distance from a starting point in its plane. It is further shown that Isbell's solution is a limiting case of the problem posed as the radius of the circle tends to infinity. The problem appears to be of some practical significance, since it is equivalent to that of searching for an object a given distance away which will be spotted when we get sufficiently close — that is, within a specific radius."

39. Gross, O., A SEARCH PROBLEM DUE TO BELLMAN, RAND Corporation, RM-1603 (AD 87 962), 8 pp., 12 Sep 1955.

Category: Geometric Search Patterns

"R. Bellman has proposed the following problem: 'Suppose that we are lost in a forest. The dimensions and shape of the forest are assumed known, but we do not know our position in the forest. What path do we pursue to minimize the time required to get out of the forest?'" This paper considers minimax solutions to the cases of several figures: circle, equilateral triangle, several "keyhole" shapes, and the infinite strip of unit width. Solutions are obtained for: circle, keyholes, and strip.

40. Gumacos, Constantine, ANALYSIS OF AN OPTIMUM SYNC SEARCH PROCEDURE, IEEE Transactions on Communications Systems, CS-11:1, pp. 89-99, Mar 1963.

Category: Allocation of Effort

"Synchronization of communications receivers can require lengthy sync search procedures. In order to establish theoretical guideposts for evaluating synchronization systems, an idealized model is assumed in which: (1) Sync exists in one, and only one, of a large number of discrete time cells, (2) the a priori probability distribution of sync position is known and (3) samples taken from any time cell are normally distributed.

"An optimum search procedure samples at any instant the cell most likely to contain sync based on all previous samples. This minimizes the average synchronization time, or, for a fixed synchronization time, it minimizes the probability of missing sync.

"The minimum theoretical average sync time is obtained for Gaussian and uniform a priori distributions. For a fixed synchronization time, the minimum theoretical probability of error is given as a function of sync time for the Gaussian and uniform a priori distributions.

"An equation for the average search time for a practical nonoptimum search procedure is derived which can be applied to any symmetric unimodal a priori distribution. For a Gaussian a priori distribution the expected value of search time for this procedure converges rapidly to optimum."

41. Hunt, J. A., THE OPTIMIZATION OF SATELLITE RECONNAISSANCE BY THE APPLICATION OF DYNAMIC PROGRAMMING TECHNIQUES, MITRE Corporation, Bedford, Mass., ESD-TDR-63-168 (AD 402 810), 9 pp., Apr 1963.

Category: Miscellaneous

Discusses the problem of a reconnaissance system with limited storage capability (i.e., there is more target object data available than there is storage capacity). Question is what targets should be observed and data recorded. Dynamic programming solution considers the situation with variable value of targets and variable quality of recording available on each target on a given mission.

42. Isaacs, Rufus, DIFFERENTIAL GAMES, John Wiley & Sons, Inc., New York, pp. 336-337, 1965.

Category: Game Theory

Geometric Search Patterns

Two search games are studied. In the first, "when the hidden objects are numerous and immobile, the time to find them (payoff) is nearly independent of the searcher's strategy as long as no effort is wasted researching territory already scouted." The second formulation considers mobile "hiders." It is conjectured that the "details of the randomization are unimportant, but certain basic parameters, such as the hider's speed are not... . There appear to be grounds for an approximate theory."

43. Isbell, J. R., AN OPTIMAL SEARCH PATTERN, Naval Research Logistics Quarterly, 4:4, pp. 357-359, Dec 1957.

Category: Geometric Search Patterns

"The problem of minimax-time search in a plane for a line at known distance is solved by a path made up of line segments and circular arcs... . A parallel strip problem [search for one of two parallel lines starting between the lines] is partially analyzed... .

It may be noted that no systematic method for such problems is available. A solution is typically piecewise smooth and the main difficulty is in determining how many pieces there are and what roles they play."

44. Johnson, Selmer M., A SEARCH GAME, Advances in Game Theory (eds.) M. Dresher, L. S. Shapley, and A. W. Tucker, Princeton University Press, Princeton, N. J., pp. 39-48, 1964.

Category: Game Theory

"Blue chooses h , an integer, from the set of integers 1 to n (a region to hide). Red guesses an integer from 1 to n , is told whether he is too high or too low. Red repeats until he guesses h . The payoff to Blue is one unit for each guess by Red (including the first guess h). Optimal strategies and game value for n equal to or less than 11 are given.

45. Karchere, Alvin, and Francis P. Hoerber, COMBAT PROBLEMS, WEAPONS SYSTEMS, AND THE THEORY OF ALLOCATION, Journal of Operations Research, 1:5, pp. 286-302, Nov 1953.

Category: Allocation of Effort

"This paper will ... attempt to establish criteria for three types of decision: (1) Given the combat situation, which weapon systems should the army use? (2) In what proportions should the various weapon systems be used? (3) Given the proportions of the weapon systems, how many of them should be employed in the given combat situation?"

This paper closes with an extensive note comparing Karchere and Hoerber's results with Koopman's [item 51]. The correlation between the two and the consistency of results are pointed out.

46. Kimball, George E., SIMPLIFIED THEORY OF SECTOR SEARCH, Operations Evaluation Group, Office of Chief of Naval Operations, Washington, D.C., OEG Study No. 282 (ATI 28 836), 9 pp., 28 June 1946a.

Category: Geometric Search Patterns

"Sector search is a scouting operation in which the scouting craft operates from a single fixed base of operations with the object of preventing the undetected approach of enemy craft. The search is assumed to be over an area of the shape of a circular sector, with the center of the circle at the base of operations. It is assumed that the enemy will approach on a course toward the base of operations, although the probability of detection is greater otherwise. The search craft are assumed to perform missions which proceed from the base to a certain distance, patrol an arc at this distance, and return to the base. As the enemy approaches the base, the probability that he has been detected will increase. The fundamental equations for this theory are derived, and an example problem is calculated."

47. Kimball, G. E., THE IDEAL SEARCH THEOREM, Operations Evaluation Group, Office of the Chief of Naval Operations, Washington, D.C., OEG Study No. 297 (ATI 28 831), 4 pp., 23 Oct 1946b.

Category: Geometric Search Patterns
Measures of Performance

This theorem provides an upper bound for the probability of detection against which the performance of a trial plan can be compared.

48. Kimball, George, DETECTION AND TRACKING AS A MARKOV PROCESS, NATO A.S.W. Meeting, Sienna, Italy, 1963.

Category: Miscellaneous

(Author was unable to locate a copy of this paper.)

49. Kobzarev, U. B. and A. E. Basharinov, ON THE EFFECTIVENESS OF SEARCH ALGORITHMS BASED ON SAMPLES OF CONTROLLED DURATION (Sequential Detection), Radiotekhnika i Elektronika, VI:9, pp. 1411-1419, Sep 1961, Translated by L. E. Brennan, RAND Corporation, RM-2953-PR (AD 270 127), 18 pp., Dec 1961.

Category: Measures of Performance

"Search procedures are considered which employ trial steps (samples) of controlled duration. Indices are defined which characterize the effectiveness of these controlled search procedures. The influence of the capacity (i.e., variety of forms of signal) on these indices is estimated. . . . We will consider three methods of controlling sample duration, i.e., three procedures for sequential search: (a) the use of statistical sequential analysis; (b) the use of grouped sequential procedures; and (c) selection of some elements for further analysis."

50. Koopman, Bernard Osgood, SEARCH AND SCREENING, Operations Evaluation Group, Office of the Chief of Naval Operations, Washington, D.C., OEG Report No. 56 (ATI 64 627), 172 pp., 1946.

Category: Allocation of Effort

Geometric Search Patterns

Measures of Performance

This is the classic original work on search theory. It represents a summary of the work done at Columbia University during World War II for the Navy.

Chapter 1, "Position, Motion, and Random Encounter," was published in Operations Research, 4:3, pp. 324-346, June, 1956. Chapter 2, "Target Detection," was published in OR 4:5, pp. 503-531, October, 1956. Chapter 3, "The Distribution of Searching Effort," was rewritten and published in OR, 5:5, pp. 613-626, October, 1957. Other chapters (not published elsewhere) are "Visual Detection," "Radar Detection," "Sonar Detection," "The Search for Targets in Transit," "Sonar Screens," and "Aerial Escorts."

51. Koopman, Bernard O., THE OPTIMUM DISTRIBUTION OF EFFORT, Journal of Operations Research, 1:2, pp. 52-63, Feb 1953.

Category: Allocation of Effort

"When a limited amount of effort is available for the performance of two related tasks, the practical question of how it is to be divided between them in order to obtain the best over-all result is one which constantly arises in operations research. . . . The present treatment is based on three general assumptions: First, it is assumed that effort can be given a numerical measure, appropriate to the particular situation considered; and that the effect, or degree to which the effort accomplishes its object, can also be expressed numerically in appropriate units. Secondly, it is assumed that the effects are additive in

the sense that the over-all effect, when effort is exerted on the two tasks, is the sum of the effect produced on one of them plus that produced on the other. Thirdly, it is assumed that the effect produced when x units of effort are exerted on a particular task depends on the task and the effort x ."

(See a note on this paper in the article by Karchere and Hoeber [item 45].)

52. Koopman, Bernard O., THE DISTRIBUTION OF SEARCHING EFFORT, Third National Meeting of the ORSA, Boston, Mass., 23-24 Nov 1953, Invited paper, Abstract in Operations Research, 2:1, p. 77, Feb 1954.

Category: Allocation of Effort

"The continuous probability distribution of an object being searched for is known before the start of the search, and the probability of finding it, if it is in any given small region, obeys an exponential saturation law in terms of the amount of search applied to this region. The problem is to distribute the search density so as to maximize the probability of finding the object. This is the prototype of a class of problems of finding the optimum distribution of effort among a continuum of different tasks. The irregular problem in the calculus of variations to which the formulation leads is solved. It is shown that in some regions no search is warranted and a quantitative criterion for this is given. In the region where searching should occur, a logarithmic law for its intensity is derived."

The material in this paper appears to be essentially the same as that appearing in Operations Research in 1956 [see items 53, 54, and 55].

53. Koopman, Bernard O., THE THEORY OF SEARCH: I. KINEMATIC BASES, Operations Research, 4:3, pp. 324-346, June 1956a.

Category: Geometric Search Patterns
Miscellaneous

"The kinematic bases, involving the positions, geometrical configurations, and motions in the searchers and targets, with particular reference to the statistics of their contacts and the probabilities of their reaching various specified relative positions."

54. Koopman, Bernard O., THE THEORY OF SEARCH: II. TARGET DETECTION, Operations Research, 4:5, pp. 503-531, Oct 1956b.

Category: Miscellaneous

"The present paper discusses the uncertainties inherent in the act of detection under various specific conditions of contact. In the course of the discussion a body of methods for applying probability to problems of detection is developed. It must be emphasized, however, that these methods are conditioned by the particular situation in the case of visual detection because the different elementary acts of looking or 'glimpses' are essentially independent trials."

55. Koopman, Bernard O., THE THEORY OF SEARCH: III. THE OPTIMUM DISTRIBUTION OF SEARCHING EFFORT, Operations Research, 5:5, pp. 613-626, Oct 1957.

Category: Allocation of Effort
Geometric Search Patterns

"Suppose that an object is in an unknown position, but that its probabilities of being in the various possible positions are known. Suppose, further, that a limited total amount of

searching effort (or time) is available. Assume, finally, that the law of detection is known, telling the chance of finding the object when a given amount of search is carried out in its vicinity. The practical problem is to find the optimum manner of distributing the available searching effort: the one which maximizes the chance of finding the object."

56. Koopman, Bernard O., *SEARCH*, Appearing in *NOTES ON OPERATIONS RESEARCH* 1959, pp. 40-83, Technology Press, MIT, 1959.

Category: Allocation of Effort

Geometric Search Patterns

"Kinematical definitions. Probability considerations. Range and region of approach. Target detection. Lateral range and sweep rate. Distribution of search effort."

Essentially the same paper published in three parts in *Operations Research* [see items 53, 54, and 55].

57. Langendorf, Patricia M., *THE PHILOSOPHY OF THE GENERAL PROBLEM OF SEARCH AND DETECTION*, Rome Air Development Center, ARDC, USAF, Griffiss AFB, Rome, N.Y., RADC-TN-59-130 (AD 213 583), 7 pp., May 1959.

Category: General Discussion

"A study was conducted to discover the way a detection system should operate to maximize detection in the absence of information regarding the signals to be detected. The results of this study indicate that the best methods of search can be developed for any transmitter without time cycle, or one whose time cycle is known. If a transmitter is operating for less time than the receiver requires for a search cycle, the maximum probability of detecting this transmitter is the ratio of duration of transmission to length of search cycle."

58. McDonald, A. M. C., J. G. Fergusson, and R. W. Elliott, *THEORY OF SEARCH*, In *SOME TECHNIQUES OF OPERATIONAL RESEARCH*, English Universities Press, London, Chapter 8, pp. 143-153, 1962.

Category: General Discussion

Allocation of Effort

Contains general discussion of the problem, although quite abbreviated. Koopman's work on distribution of effort [item 55] is discussed, as is Engel's paper [item 28] on clustering in surveys. A very clear explanation of Koopman's results is given. Unfortunately, this book is difficult to locate in the United States.

59. MacQueen, J., and R. G. Miller, Jr., *OPTIMAL PERSISTENCE POLICIES*, *Operations Research*, 8:3, pp. 362-380, May-June 1960.

Category: Allocation of Effort

"This paper deals with the problem of whether or not a search activity should be started and, if started, whether or not it should be continued. This problem suggests a model that is described. The model gives rise to a general functional equation for which existence and uniqueness conditions are given. Several examples are discussed, solutions to the specific functional equations appropriate to the examples are given, and the optimal policies are characterized.

"The persistence problem is the problem of determining whether or not a search activity should be started and, if it is started, how long it should be continued."

60. Matula, David, A PERIODIC OPTIMAL SEARCH, American Math. Monthly, 71:1, pp. 15-21, Jan 1964.

Category: Allocation of Effort

"The problem is to find a program $\pi = (\pi(1), \pi(2), \dots)$, i.e., a sequence of locations to be searched such that the expected cost $V(\pi)$, of finding the object is minimal. Note that, in general, to be successful, each location must be searched infinitely often.

"A program is called ultimately periodic if $\pi(j+\theta) = \pi(j)$ for all $j > T$, where T denotes the length of the transient phase and θ the length of the period. Our major result... yields the conditions for the existence of an ultimately periodic optimal program and also determines the minimal period and the minimal transient length."

61. Mela, Donald F., INFORMATION THEORY AND SEARCH THEORY AS SPECIAL CASES OF DECISION THEORY, Operations Research, 9:6, pp. 907-909, Nov-Dec 1961.

Category: Measures of Performance

Presents the contention that "the connection between the information-theory approach and the search-theory approach is tenuous" without exploring reasons for using information-theory approach. Does point out that distribution of effort to maximize change in entropy is different from that to maximize "detection probability." "Probability of correct commitment" was also mentioned. Only two examples (2- and 3-cell equally probable) are given to support these results. [Gives as the only previous reference on this topic Paul B. Coggins, "Application of Information Theory to Search," Memorandum for Director of Research, Operations Evaluation Group (L0) 1975-52, 22 Oct 1952 (unpublished).]

62. Morse, Philip M., MATHEMATICAL PROBLEMS IN OPERATIONS RESEARCH, Bulletin of the American Mathematics Society, 54:7 (No. 586), pp. 602-621, July 1948.

Category: General Discussion

A speech on the general topic of mathematics in operations research. Fifteen pages of the article are devoted to "The search problem." What is presented is a very clear general discussion of the problem based primarily on Koopman's work [item 50].

63. Morse, Philip M. and George E. Kimball, METHODS OF OPERATION RESEARCH, The Massachusetts Institute of Technology Press, Cambridge, Mass., 1951.

Category: General Discussion

This early classic in operation research literature discusses several problems (not of very general applicability) that were studied analytically using Koopman's results [see item 50].

64. Neuts, Marcel F., A MULTISTAGE SEARCH GAME, Journal of SIAM, 11:2, pp. 502-507, June 1963.

Category: Allocation of Effort

Game Theory

"Player I hides an object, in one of N boxes labeled from 1 to N . His opponent, Player II, has to search for the object by successive examinations of the boxes. An examination of the i th box, $i = 1, \dots, N$, can be performed at a cost $t_i > 0$ each time and there is a probability p_i , $0 < p_i \leq 1$, of finding the object given that the right box is searched.

"Upon finding the item Player II receives a reward of $a \geq 0$. We shall solve the game explicitly under the assumption that Player II is restricted to the use of stationary strategies and discuss the solution of Bellman's functional equations for the Bayes risk in the nonstationary case. A stationary strategy for Player II is a probability distribution over the integers $1, \dots, N$ which is chosen once and for all and which serves to determine the box which should be examined at every search. In essence the use of a stationary strategy amounts to dropping all information obtained from previous searches (a memoryless Player II).

"Let δ , $0 < \delta \leq 1$, stand for a discount factor which is applied to losses at future stages of the game. When $0 < \delta < 1$ the loss-function for Player II is bounded and the nonstationary case may be solved using dynamic programming methods. The case $\delta = 1$, in which future losses are not discounted, is more involved but it can be studied using the results of § 2 on the stationary minimax strategies."

65. Nichols, R. E. and W. M. Whisler, A TECHNIQUE FOR ANALYSIS OF INTERMITTENT SEARCH OPERATIONS APPLICABLE TO ASW, Boeing Airplane Co., Seattle 24, Wash., D2-10868, 31 pp., 29 May 1961.

Category: Geometric Search Patterns

Search operations are conducted intermittently with the escort screen moving between searching periods. No searching can be conducted while the escort screen is moving.

"The purposes of this writing are to show:

- (1) A method to normalize the variables involved in intermittent sonar search.
- (2) Some parametric analyses made possible by the use of that method."

Two search and movement patterns are analyzed — zig-zag and double line search.

66. Norris, R. C., STUDIES IN SEARCH FOR A CONSCIOUS EVADER, Lincoln Lab., M.I.T., Tech Report No. 279 (AD 294 832), 134 pp., 14 Sep 1962.

Category: Allocation of Effort
Game Theory

"This paper considers a search problem in which the search is directed against a conscious evader or an object controlled by a conscious evader. It is a two-person, zero-sum game called a search evasion game. Although the searcher cannot observe any of the evader's actions, the evader can observe the searcher's and can capitalize on errors that he makes.

"At the beginning of the game, the evader hides in one of several boxes. The search process consists of a sequence of looks into the various boxes until the evader is found. Each look into a given box takes a fixed amount of time. If the searcher looks into the box in which the evader is located, he will find the evader with a certain probability — the detection probability associated with the box in question. A particular evasion device is assumed: the evader can move from one box to another between looks. A cost is usually associated with such a move."

67. Novosad, Robert S., SEARCH PROBLEMS AND INFORMATION THEORY, Operations Research Department, Martin - Denver, Colo., Working Paper #64, 5 June 1961.

Category: Measures of Performance

Search strategies may be optimized with regard to various objectives. This paper considers two such search objectives:

1. The probability of detection (i.e., viewing the target) shall be maximized.
2. The added information which is expected from the search shall be maximized or, what amounts to the same thing, the uncertainty which can be expected at the conclusion of the search shall be minimized.

A simple example will show that type 1 and type 2 objectives can lead to different search strategies.

68. Pollock, Stephen M., OPTIMAL SEQUENTIAL STRATEGIES FOR TWO REGION SEARCH WHEN EFFORT IS QUANTIZED, Operations Research Center, MIT., Cambridge, Mass., Interim Tech Report No. 14 (AD 238 662), 67 pp., May 1960.

Category: Allocation of Effort

"We treat here the case of a single searcher when the minimal unit of effort allocatable (a "look") is positive, with a corresponding discrete detection probability q_1 . The optimal sequential strategies of searching are then explicitly found for the two-region case, using a dynamic programming approach.

"Comparison with the continuous case shows an obvious asymptotic equivalence. Further, although the optimal searching procedures and effort expended otherwise differ from the continuous case, under certain restrictions the procedures merely provide an explicit scheme for adapting the continuous theory when of practical necessity search effort must be quantized."

69. Pollock, Stephen Michael, SEQUENTIAL SEARCH AND DETECTION, Operations Research Center, MIT., Cambridge, Mass., Tech Report No. 5 [Contract Nonr-3963(06)], 131 pp., May 1964.

Category: Allocation of Effort

"The objective of this study has been to obtain and evaluate strategies to be used in certain general search situations. These strategies minimize the expected cost of search and resulting decisions and are sequential in the sense that a decision at any time is dependent upon what has been observed up to that time.

"The first situation studied leads to the formulation of a minimum expected cost sequential hypothesis test. The target is either present in the region of interest with a priori probability P , or not with probability $1 - P$. Knowing the value of P , at fixed intervals of time the searcher must either make a terminal decision (i.e., decide that the target is present, or not present) or make a measurement of a random variable that has a probability density function which depends upon whether or not the target is present. A cost structure is given which assigns costs to wrong terminal decisions, as well as a cost (which depends upon whether or not the target is present) for the taking of a measurement. The sequential strategy and resultant minimum cost are derived by solving a functional equation of the dynamic programming type. The relation between this strategy and the Wald sequential probability ratio test is discussed. The minimum cost of the strategy is compared with the cost of an often used non-sequential strategy as well as a class of sub-optimal sequential strategies that involve threshold observations.

"The second part of this study involves a situation in which it is assumed that the target arrives at some random time (the "raid-recognition" problem). A cost structure is given which assigns a cost to deciding the target has arrived when in fact it hasn't, and also assigns a cost proportional to the time between arrival of the target and the decision that it has arrived. Again observations of a random variable related to the presence of the target are available as an alternative to making such a decision. The sequential strategy and resulting minimum cost are again obtained by means of a functional equation. An additional result is the formulation of a System Operating Characteristic that is used for this randomly arriving target model in a way similar to the use of the Receiver Operating Characteristic for the hypothesis test model."

70. Posner, Edward C., OPTIMAL SEARCH PROCEDURES, IEEE Transactions on Information Theory, IT-9:3, pp. 157-160, July 1963.

Category: Allocation of Effort

Geometric Search Patterns

"This paper sets up a restricted class of search procedures for a satellite lost in a region of the sky. The satellite must be found by a radar search. The procedures under consideration allow the use of a preliminary search, which may be done with a wider beam than is required for the final search. The purpose of the preliminary search is to obtain a ranking of the various portions of the sky, so that the final search can examine the more likely regions of the sky first. It is shown that a preliminary search can reduce the expected search time, with no matter how wide a beam it is carried out. It is also shown that the preliminary search with the narrowest possible beam is best."

71. Potter, Norman S., THE OPTIMIZATION OF ASTRONAUTICAL VEHICLE DETECTION SYSTEMS THROUGH THE APPLICATION OF SEARCH THEORY, Proc. IRE, 48:4, pp. 541-553, Apr 1960 (Space Electronics Issue).

Category: Miscellaneous

Discusses a very specific problem in which some of the results of search theory have been applied. The kinematic basis of the problem (detecting meteors from a satellite for instance) is discussed and some quantitative results are obtained.

72. Potter, Norman S., PROGRAMMED SEARCH IN ADAPTIVE SYSTEMS, IRE Trans. on Mil. Elec., MIL-5:4, pp. 362-369, Oct 1961.

Category: Allocation of Effort

"An investigation is conducted of the programming of search by discrete data systems over a space volume. The distribution of search effort which leads to the greatest attainable information rate on a contact, or probability of its retention by maximizing the probability of a positive interrogation within some designated time interval is determined. An estimate of p , the relative frequency of positive interrogations of an individual contact, is utilized as a basis for adjustment of the sampling rate in accordance with results of the search program optimization study. It is shown that in the case of Rayleigh signal sources, the rate of convergence to a stable estimate of p is greatest if a search program characterized by a rapid sampling rate and a consequent low probability of detection on the individual trial is employed."

73. Scott, Kenneth Robert, OPTIMAL MULTI-REGION DISCRETE SEARCH UNDER LINEAR SEARCH COST, Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass., Master of Science Thesis, 18 Jan 1963.

Category: Allocation of Effort

"The problem of search among a discrete number of regions was studied by means of a reward-cost model. In this model a searcher examines regions sequentially to find an object which, if found, yields a reward. There is a cost c_i assigned to each look taken in the i th region. Furthermore, the a priori probability of the object being in region i , p_i , and the probability of detecting the object on a look if the object is present, a_i , is known to the searcher. The quantities a_i and c_i are allowed to be constant or to be functions of the number of looks taken in region i . Policies maximizing the rate-of-return and the expected value are sought.

"It has been found that a class of policies may be specified such that one member of this class maximizes the rate-of-return and one member (perhaps different) maximizes the expected value. If at any stage the number of looks remaining to be taken in the optimal policy is infinite (or may be approximated as such) the particular policy that maximizes both the rate-of-return and the expected value may be specified; the searcher looks in the region for which pa/c is greatest. It will be noted that the single policy (when it can be used) is the same as that policy previously found by Black [item 10] and Pollock [item 69] to minimize the cost of a search among discrete regions."

74. Sherman, Seymour, TOTAL RECONNAISSANCE WITH TOTAL COUNTERMEASURES, RAND Corporation, 1700 Main Street, Santa Monica, Calif., RM-202, 18 pp., 5 Aug 1949.

Category: Game Theory

"Introduction: We have a game, where only a finite set of pure strategies is available to each player, and consider how the pay-off and desirable strategies are affected by the possibilities of total reconnaissance (at a given cost) and total countermeasures to reconnaissance (at a given cost). The model is a simplified one.

"Results: We determine

(1) an easily computed assured gain for the first player (who may reconnoiter); this easy computation involves merely the substitution in a formula which uses the cost of reconnaissance, the cost of countermeasures, and the value of the original game (without reconnaissance);

(2) an easily computed strategy which assures the first player the gain cited above;

(3) a sufficient condition that the second player can limit his loss to the amount cited above."

75. Zahl, Samuel, AN ALLOCATION PROBLEM WITH APPLICATIONS TO OPERATIONS RESEARCH AND STATISTICS, Operations Research, 10:3, May-June 1963.

Category: Allocation of Effort

"A considerable number of problems in operations research and statistics have the following form: maximize $\int f(x, y(x)) dx$ subject to $\int g(x, y(x)) dx = \text{constant}$ with respect to bounded $y(x)$. We give a necessary and sufficient condition for a maximizing function under fairly weak restrictions and prove its existence. The solution is applied to a general version of B. O. Koopman's search problem, and to the Neyman, Pearson lemma of

statistics. We also show that in the discrete version of this problem, where x is replaced by an index and sums replace integrals, our condition is sufficient but not necessary and give, as illustration of sufficiency, a solution to an assignment problem."

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